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BEHAVIOR OF A FLOATING ELASTIC PLATE DURING VIBRATIONS OF A BOTTOM SEGMENT

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The Wiener–Hopf technique is used to obtain an analytical solution for the problem of vibrations of a floating semi-infinite elastic plate due to earthquake-induced vibrations of a bottom segment. An explicit solution is obtained ignoring the inertial term. The surface-wave amplitudes and ice-plate deflection are studied numerically as functions of the frequency and position of the vibrating bottom segment, ice thickness, and fluid depth.

Key words: *surface waves, flexural-gravity waves, elastic thin plate, Wiener–Hopf technique.*

Introduction. The problem of the hydroelastic behavior of floating elastic plates has been studied earlier as applied to an ice cover [1–3]. At present, interest in this problem has increased because of projects on the construction of floating airfields, artificial islands, and floating platforms of various applications. Because of the huge sizes of such objects, the similarity parameters are difficult to satisfy in experiments; therefore, numerical modeling plays a great role in their study.

The problem of surface-wave diffraction on a floating elastic plate has been studied fairly extensively. Less attention has been given to the dynamic behavior of a floating plate under external loading (a review [4]) and plate behavior during earthquakes [5]. In [5], the bottom is modeled by a homogeneous elastic medium (half-space), in which longitudinal and transverse waves propagate from the earthquake epicenter, and the fluid is considered compressible and imponderable. The present paper studies the behavior of a semi-infinite elastic plate floating on the surface of an incompressible ponderable fluid under specified periodic vibrations of the bottom. An analytical solution of this problem in a plane formulation is constructed using the Wiener–Hopf technique.

1. Formulation of the Problem. It is assumed that the fluid is perfect and incompressible, the fluid depth is H_0, and its flow is vortex-free. The plate is a semiplane of constant thickness h. The plate vibrations are caused by time-periodic vibrations of the bottom. The left edge of the plate is taken to be the origin of Cartesian coordinates Oxy. The plate thickness is assumed to be much smaller than length of the waves propagating in the plate. We use the model of thin plates. The plate draft is ignored, and the boundary conditions are extended to the unperturbed water surface.

The fluid-velocity potential φ satisfies the Laplace equation and the boundary conditions

$$\begin{gathered}\Delta\varphi = 0 \qquad (-H_0 < y < 0),\\ \varphi_y = w_t \quad (y = 0, -H_0), \qquad w(x, -H_0, t) = u(x)\,\mathrm{e}^{-i\omega t},\\ D\,\frac{\partial^4 w}{\partial x^4} + \rho_0 h\,\frac{\partial^2 w}{\partial t^2} = p \qquad (y = 0, \quad x > 0),\\ p = 0 \quad (y = 0,\ x < 0), \qquad p = -\rho(\varphi_t + gw).\end{gathered} \tag{1.1}$$

Here w is the vertical displacement of the upper surface of the fluid (plate), p is the hydrodynamic pressure, g is the acceleration of gravity, D is the flexural rigidity of the plate, ρ and ρ_0 are the densities of the fluid and plate,

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t is time, ω is the vibration frequency of the bottom, and $u(x)$ is the amplitude of bottom displacements. At the edge of the plate, the moment and shear force should vanish:

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^3 w}{\partial x^3} = 0 \qquad (y = 0, \quad x = 0). \tag{1.2}$$

We first consider the case of a point source on the bottom: $u(x) = u_0\delta(x - x_0)$. The time dependence of all functions is expressed by the factor $\mathrm{e}^{-i\omega t}$. Let us enter the characteristic length $l = g/\omega^2$ and the dimensionless variables

$$x' = \frac{x}{l}, \quad y' = \frac{y}{l}, \quad x_* = \frac{x_0}{l}, \quad H = \frac{H_0}{l}, \quad \varphi' = \frac{\omega\varphi}{gu_0}, \quad w' = \frac{w}{u_0}, \quad t' = \omega t$$

(below, the primes are omitted). Writing the potential as $\varphi = \phi\,\mathrm{e}^{-it}$, from (1.1) and (1.2), we obtain the following boundary-value problem for ϕ:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad (-H < y < 0);$$

$$\frac{\partial \phi}{\partial y} = -i\delta(x - x_*) \qquad (y = -H); \tag{1.3}$$

$$\frac{\partial \phi}{\partial y} - \phi = 0 \qquad (y = 0, \quad x < 0); \tag{1.4}$$

$$\Big(\beta\,\frac{\partial^4}{\partial x^4} + 1 - d\Big)\frac{\partial \phi}{\partial y} - \phi = 0 \quad (y = 0,\ x > 0), \qquad \beta = \frac{D}{\rho g l^4}, \quad d = \frac{\rho_0 h}{\rho l}; \tag{1.5}$$

$$\frac{\partial^2}{\partial x^2}\,\frac{\partial \phi}{\partial y} = \frac{\partial^3}{\partial x^3}\,\frac{\partial \phi}{\partial y} = 0 \qquad (y = 0, \quad x = 0). \tag{1.6}$$

Here H, x_*, β, and d are the dimensionless parameters of the problem: the fluid depth, the center of the vibrating segment, the reduced rigidity of the plate, and the plate draft. In addition, the radiation conditions for $|x| \to \infty$ and the regularity conditions near the edges (local boundedness of the energy) should be satisfied.

2. Integral Equations. The solution of the problem is constructed using the Wiener–Hopf technique in the Jones interpretation [6]. Let us consider the following functions of the complex variable α:

$$\Phi_+(\alpha, y) = \int_0^\infty \mathrm{e}^{i\alpha x}\,\phi(x, y)\,dx, \qquad \Phi_-(\alpha, y) = \int_{-\infty}^0 \mathrm{e}^{i\alpha x}\,\phi(x, y)\,dx,$$

$$\Phi(\alpha, y) = \Phi_-(\alpha, y) + \Phi_+(\alpha, y). \tag{2.1}$$

The function $\Phi_+(\alpha, y)$ is defined in the upper semiplane $\operatorname{Im}\alpha > 0$, and the function $\Phi_-(\alpha, y)$ in the lower semiplane $\operatorname{Im}\alpha < 0$. By analytic continuation, these functions can be defined on the entire complex plane. The function $\Phi(\alpha, y)$ is the Fourier transform of the function $\phi(x, y)$ and satisfies the equation

$$\frac{\partial^2 \Phi}{\partial y^2} - \alpha^2\Phi = 0.$$

On the bottom, condition (1.3) becomes

$$\frac{\partial \Phi}{\partial y}(\alpha, -H) = -i\,\mathrm{e}^{i\alpha x_*}\,. \tag{2.2}$$

The solution of this equation sought in the form

$$\Phi(\alpha, y) = C(\alpha)Z(\alpha, y) + S(\alpha)\sinh\,(\alpha(y + H)), \qquad Z(\alpha, y) = \cosh\,(\alpha(y + H))/\cosh\,(\alpha H).$$

From (2.2), we obtain $S(\alpha) = -i\,\mathrm{e}^{i\alpha x_*}/\alpha$. Then,

$$\Phi(\alpha, y) = C(\alpha)Z(\alpha, y) - i\,\mathrm{e}^{i\alpha x_*}\sinh\,(\alpha(y + H))/\alpha. \tag{2.3}$$

We denote by $D_\pm(\alpha)$ integrals of type (2.1) in which the integrand ϕ is replaced by the left side of condition (1.4) and by $F_\pm(\alpha)$ similar expressions in which the integrand is the left side of expression (1.5). Let us introduce the functions

$$D(\alpha) = D_-(\alpha) + D_+(\alpha), \qquad F(\alpha) = F_-(\alpha) + F_+(\alpha),$$

which are the Fourier transforms of dispersion functions, which will be understood in the sense of generalized functions [7]:

$$D(\alpha) = \frac{\partial \Phi}{\partial y}(\alpha, 0) - \Phi(\alpha, 0), \qquad F(\alpha) = (\beta\alpha^4 + 1 - d)\,\frac{\partial \Phi}{\partial y}(\alpha, 0) - \Phi(\alpha, 0).$$

From the boundary conditions (1.4) and (1.5), we have

$$D_-(\alpha) = 0, \qquad F_+(\alpha) = 0.$$

Then,

$$D_+(\alpha) = D(\alpha) = C(\alpha)K_1(\alpha) - i\,e^{i\alpha x_*}[\cosh\,(\alpha H) - \sinh\,(\alpha H)/\alpha]; \tag{2.4}$$

$$F_-(\alpha) = F(\alpha) = C(\alpha)K_2(\alpha) - i\,e^{i\alpha x_*}[(\beta\alpha^4 + 1 - d)\cosh\,(\alpha H) - \sinh\,(\alpha H)/\alpha]. \tag{2.5}$$

Here $K_1(\alpha) = \alpha \tanh\,(\alpha H) - 1$ and $K_2(\alpha) = (\beta\alpha^4 + 1 - d)\alpha \tanh\,(\alpha H) - 1$ are the dispersion functions for the free-surface fluid and the fluid under the plate. Both dispersion functions are even. The dispersion relation on the free surface $K_1(\alpha) = 0$ has two real roots $\pm\gamma$ and a countable set of purely imaginary roots $\pm\gamma_n$ $(n = 1, 2, \ldots)$ which are symmetric about the real axis [8]; $\gamma_n \to in\pi/H$ as $n \to \infty$.

The dispersion relation under the plate $K_2(\alpha) = 0$ has two real roots $\pm\alpha_0$, a countable set of purely imaginary roots $\pm\alpha_n$ $(n = 1, 2, \ldots)$ symmetric about the real axis, and four complex roots which are symmetric about the real and imaginary axes [8]. We denote by α_{-1} the root lying in the first quadrant and by α_{-2} the root in the second quadrant; $\alpha_n \to in\pi/H$ at $n \to \infty$.

The real roots of the dispersion relations determine the propagating surface and flexural-gravity waves, and the remaining roots determine the edge waves, which damp exponentially away from the perturbation source.

We examine the behavior of the functions $\Phi_\pm(\alpha, y)$. For $x \to -\infty$, the potential ϕ represents a wave of the form $R e^{-i\gamma x}$ and a set of exponentially damped waves. The least damped wave corresponds to the root γ_1. Therefore, $\Phi_-(\alpha, y)$ is analytic in the semiplane $\operatorname{Im}\alpha < |\gamma_1|$ except for the pole at $\alpha = \gamma$. For $x \to \infty$, the potential ϕ represents a wave of the form $T\,e^{i\alpha_0 x}$ and a set of exponentially damped modes. Therefore, the function $\Phi_+(\alpha, y)$ is analytic in the semiplane $\{\operatorname{Im}\alpha > -c\}$ except for the pole at the point $\alpha = -\alpha_0$, where $c = \min\{\operatorname{Im}(\alpha_1), \operatorname{Im}(\alpha_{-1})\}$.

Eliminating $C(\alpha)$ from relations (2.4) and (2.5), we obtain the equation

$$[F_-(\alpha) + i\,e^{i\alpha x_*}[(\beta\alpha^4 + 1 - d)\cosh\,(\alpha H) - \sinh\,(\alpha H)/\alpha]]K(\alpha)$$

$$= D_+(\alpha) + i\,e^{i\alpha x_*}[\cosh\,(\alpha H) - \sinh\,(\alpha H)/\alpha], \tag{2.6}$$

$$K(\alpha) = K_1(\alpha)/K_2(\alpha).$$

According to the Wiener–Hopf technique, it is necessary to factorize the function $K(\alpha)$, i.e., to write it as

$$K(\alpha) = K_+(\alpha)K_-(\alpha),$$

where the functions $K_\pm(\alpha)$ are regular in the same regions as the functions $\Phi_\pm(\alpha, y)$. The function $K(\alpha)$ has zeroes and poles at the points $\pm\gamma$ and $\pm\alpha_0$, respectively, on the real axis. Therefore, we consider the analyticity regions S_+ and S_-, where S_+ is the semiplane $\operatorname{Im}\alpha > -c$ with cuts eliminating the points $-\alpha_0$ and $-\gamma$ and S_- is the semiplane $\operatorname{Im}\alpha < |\gamma_1|$ with cuts eliminating the points α_0 and γ.

Let us introduce the function

$$g(\alpha) = K(\alpha)\beta(\alpha^2 - \alpha_0^2)(\alpha^2 - \alpha_{-1}^2)(\alpha^2 - \alpha_{-2}^2)/(\alpha^2 - \gamma^2).$$

The function $g(\alpha)$ does not have zeroes on the real axis, is limited, and tends to unity at infinity. We factorize $g(\alpha)$ as follows [6]:

$$g(\alpha) = g_+(\alpha)g_-(\alpha), \qquad g_\pm(\alpha) = \exp\Big[\pm\frac{1}{2\pi i}\int\limits_{-\infty\mp i\sigma}^{\infty\mp i\sigma}\frac{\ln g(x)}{x-\alpha}\,dx\Big], \qquad \sigma < |\gamma_1|.$$

The function $K_\pm(\alpha)$ is defined by the formula

$$K_\pm(\alpha) = \frac{(\alpha\pm\gamma)g_\pm(\alpha)}{\sqrt{\beta}\,(\alpha\pm\alpha_0)(\alpha\pm\alpha_{-1})(\alpha\pm\alpha_{-2})}.$$

In this case, $K_+(\alpha) = K_-(-\alpha)$.

Dividing Eq. (2.6) by $K_+(\alpha)$ and performing some transformations, we obtain

$$F_-(\alpha)K_-(\alpha) - \frac{i\,e^{i\alpha x_*}(\beta\alpha^4 - d)}{\cosh\,(\alpha H)K_2(\alpha)K_+(\alpha)} = \frac{D_+(\alpha)}{K_+(\alpha)}. \tag{2.7}$$

Using the representation

$$\frac{e^{i\alpha x_*}(\beta\alpha^4 - d)}{\cosh\,(\alpha H)K_2(\alpha)K_+(\alpha)} = L_+(\alpha) + L_-(\alpha),$$

where

$$L_\pm(\alpha) = \pm\frac{1}{2\pi i}\int\limits_{-\infty\mp i\sigma}^{\infty\mp i\sigma}\frac{e^{i\zeta x_*}(\beta\zeta^4 - d)}{\cosh\,(\zeta H)K_2(\zeta)K_+(\zeta)(\zeta-\alpha)}\,d\zeta$$

$$= \pm\frac{1}{2\pi i}\int\limits_{-\infty\mp i\sigma}^{\infty\mp i\sigma}\frac{e^{i\zeta x_*}(\beta\zeta^4 - d)K_-(\zeta)}{\cosh\,(\zeta H)K_1(\zeta)(\zeta-\alpha)}\,d\zeta, \qquad \sigma < \min\{|\gamma_1|, c\}, \tag{2.8}$$

we write Eq. (2.7) as

$$K_-(\alpha)F_-(\alpha) - iL_-(\alpha) = D_+(\alpha)/K_+(\alpha) + iL_+(\alpha).$$

The left side of this equality contains a function analytic in the region S_-, and the right side contains a function analytic in S_+. By analytic continuation, one obtains a function analytic in the entire complex plane. According to Liouville's theorem, this function is a polynomial. The degree of the polynomial is determined by the behavior of the functions as $|\alpha| \to \infty$.

The condition of local boundedness of the energy implies that near the edge of the plate, the velocities have a singularity of order not higher than $O(r^{-\lambda})$ ($\lambda < 1$; r is the distance to the plate edge). Then, for $|\alpha| \to \infty$, the function $F_-(\alpha)$ has order not higher than $O(|\alpha|^{\lambda+3})$ and the function $D_+(\alpha)$ has order not higher $O(|\alpha|^{\lambda-1})$ [7]. At infinity, the functions $K_\pm(\alpha)$ have order $O(|\alpha|^{-2})$ because $g^\pm(\alpha) \to 1$ as $|\alpha| \to \infty$. It is easy to show that $|L_\pm(\alpha)| = O(|\alpha|^{-1})$ as $|\alpha| \to \infty$. Therefore, the degree of the polynomial is equal to unity and

$$D_+(\alpha)/K_+(\alpha) + iL_+(\alpha) = i(a + b\alpha), \tag{2.9}$$

where a and b are unknown constants, which will be determined from conditions (1.6).

Expressing $D_+(\alpha)$ from relation (2.9) and taking into account (2.3) and (2.4), we obtain

$$\Phi(\alpha, y) = iZ(\alpha, y)\{K_+(\alpha)[a + b\alpha - L_+(\alpha)] + e^{i\alpha x_*}[\cosh\,(\alpha H) - \sinh\,(\alpha H)/\alpha]\}/K_1(\alpha)$$

$$- i\,e^{i\alpha x_*}\sinh\,(\alpha(y+H))/\alpha.$$

By inverse Fourier transformation, the potential ϕ is expressed as

$$\phi(x, y) = -\frac{1}{2\pi i}\int\limits_{-\infty}^{\infty}\frac{e^{-i\alpha x}\,K_+(\alpha)[a + b\alpha - L_+(\alpha)]Z(\alpha, y)}{K_1(\alpha)}\,d\alpha$$

$$-\frac{1}{2\pi i}\int\limits_{-\infty}^{\infty}\frac{e^{-i\alpha(x-x_*)}[\cosh\,(\alpha H) - \sinh\,(\alpha H)/\alpha]Z(\alpha, y)}{K_1(\alpha)}\,d\alpha + \frac{1}{2\pi i}\int\limits_{-\infty}^{\infty}\frac{e^{-i\alpha(x-x_*)}\sinh\,(\alpha(y+H))}{\alpha}\,d\alpha. \tag{2.10}$$

The integration contour should lie entirely in the intersection of the regions S_+ and S_-. The integration contour can be chosen on the real axis so that it encircles the points α_0 and γ from below and the points $-\alpha_0$ and $-\gamma$ from above.

For the derivative of the potential on the real axis, we obtain the expression

$$\frac{\partial\varphi}{\partial y}(x,0) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{\mathrm{e}^{-i\alpha x}\, K_+(\alpha)[a+b\alpha - L_+(\alpha)]\alpha \tanh\,(\alpha H)}{K_1(\alpha)}\, d\alpha - \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{\mathrm{e}^{-i\alpha(x-x_*)}}{\cosh\,(\alpha H) K_1(\alpha)}\, d\alpha. \quad (2.11)$$

Multiplying the numerator and denominator in the first integral by $K_-(\alpha)$ and performing some transformations, we obtain the following expression for the derivative of the potential:

$$\frac{\partial\varphi}{\partial y}(x,0) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{\mathrm{e}^{-i\alpha x}[a+b\alpha + L_-(\alpha)]\alpha \tanh\,(\alpha H)}{K_-(\alpha) K_2(\alpha)}\, d\alpha - \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{\mathrm{e}^{-i\alpha(x-x_*)}}{\cosh\,(\alpha H) K_2(\alpha)}\, d\alpha. \quad (2.12)$$

The integral is calculated using residue theory. On the plate at $x > 0$, we have

$$\frac{\partial\varphi}{\partial y}(x,0) = \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j x}\, \alpha_j \tanh\,(\alpha_j H)[a - b\alpha_j + L_-(-\alpha_j)]}{K_-(-\alpha_j) K_2'(-\alpha_j)} - \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j |x-x_*|}}{\cosh\,(\alpha_j H) K_2'(\alpha_j)}. \quad (2.13)$$

From the dispersion relation under the plate, we have

$$\alpha_j \tanh\,(\alpha_j H) = -K_1(\alpha_j)/(\beta\alpha_j^4 - d).$$

Substitution of this expression into formula (2.13) and then into boundary conditions (1.6) yields the following system of second-order linear algebraic equations for the unknowns a and b:

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}. \quad (2.14)$$

According to the residue theorem, the coefficients of the system are written as

$$A_{11} = \sum_{k=1}^{4} \mathrm{res}_{z_k} \Big(\frac{\alpha^2 K_+(\alpha)}{\beta\alpha^4 - d}\Big), \qquad A_{12} = A_{21},$$

$$A_{21} = \sum_{k=1}^{4} \mathrm{res}_{z_k} \Big(\frac{\alpha^3 K_+(\alpha)}{\beta\alpha^4 - d}\Big), \qquad A_{22} = \sum_{k=1}^{4} \mathrm{res}_{z_k} \Big(\frac{\alpha^4 K_+(\alpha)}{\beta\alpha^4 - d}\Big),$$

$$C_1 = -\sum_{k=1}^{4} \mathrm{res}_{z_k} \Big(\frac{\alpha^2 K_+(\alpha) L_-(\alpha)}{\beta\alpha^4 - d}\Big), \qquad C_2 = -\sum_{k=1}^{4} \mathrm{res}_{z_k} \Big(\frac{\alpha^3 K_+(\alpha) L_-(\alpha)}{\beta\alpha^4 - d}\Big),$$

where z_k are roots of the polynomial $\beta\alpha^4 - \delta = 0$. From (2.8), we have

$$L_+(\alpha) = \begin{cases} \displaystyle\sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j x_*}(\beta\alpha_j^4 - d)}{\cosh\,(\alpha_j H) K_2'(\alpha_j) K_+(\alpha_j)(\alpha_j - \alpha)} + \frac{\mathrm{e}^{i\alpha x_*}(\beta\alpha^4 - d)}{\cosh\,(\alpha H) K_2(\alpha) K_+(\alpha)}, & x_* > 0,\ \alpha \neq \alpha_j, \\ \displaystyle -\sum_{k=0}^{\infty} \frac{\mathrm{e}^{-i\gamma_k x_*}(\beta\gamma_k^4 - d) K_+(\gamma_k)}{\cosh\,(\gamma_k H) K_1'(\gamma_k)(\gamma_k + \alpha)}, & x_* < 0, \end{cases}$$

$$L_-(\alpha) = \begin{cases} \displaystyle -\sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j x_*}(\beta\alpha_j^4 - d)}{\cosh\,(\alpha_j H) K_2'(\alpha_j) K_+(\alpha_j)(\alpha_j - \alpha)}, & x_* > 0, \\ \displaystyle \sum_{k=0}^{\infty} \frac{\mathrm{e}^{-i\gamma_k x_*}(\beta\gamma_k^4 - d) K_+(\gamma_k)}{\cosh\,(\gamma_k H) K_1'(\gamma_k)(\gamma_k + \alpha)} + \frac{\mathrm{e}^{i\alpha x_*}(\beta\alpha^4 - d) K_-(\alpha)}{\cosh\,(\alpha H) K_1(\alpha)}, & x_* < 0,\ \alpha \neq -\gamma_k. \end{cases}$$

The coefficients of the system are converted as follows:

$$A_{11} = \sum_{k=1}^{4} \frac{K_+(z_k)}{z_k}, \qquad A_{12} = A_{21} = \sum_{k=1}^{4} K_+(z_k), \qquad A_{22} = \sum_{k=1}^{4} z_k K_+(z_k),$$

$$C_1 = -\sum_{k=1}^{4} \frac{K_+(z_k)L_-(z_k)}{z_k}, \qquad C_2 = -\sum_{k=1}^{4} K_+(z_k)L_-(z_k).$$

Determining the coefficients a and b from system (2.14) and substituting them into formulas (2.10)–(2.12), we find all the required quantities.

The plate deflection is determined from (1.1) using the relation $w(x) = i\varphi_y(x, 0)$ and expression (2.13). The second term in (2.13) represents the waves propagating from the perturbation point and coincides with the expression for the deflection of an infinite plate, and the first term in (2.13) represents the wave reflected from the edge. The elevation of the free boundary $\eta(x)$ has the form

$$\eta(x) = -i\sum_{k=0}^{\infty} \frac{e^{-i\gamma_k x}\, K_+(\gamma_k)[a + b\gamma_k - L_+(\gamma_k)]}{K_1'(\gamma_k)} - i\sum_{k=0}^{\infty} \frac{e^{i\gamma_k|x-x_*|}}{\cosh\,(\gamma_k H)K_1'(\gamma_k)}.$$

For $x_* > 0$, we obtain the following formulas for the plate deflection and the free-boundary elevation:

$$w(x) = -i\sum_{j=-2}^{\infty} \frac{e^{i\alpha_j x}\,\alpha_j \tanh\,(\alpha_j H)}{K_+(\alpha_j)K_2'(\alpha_j)}\Big[a - b\alpha_j - \sum_{m=-2}^{\infty} \frac{e^{i\alpha_m x_*}(\beta\alpha_m^4 - d)}{\cosh\,(\alpha_m H)K_2'(\alpha_m)K_+(\alpha_m)(\alpha_m + \alpha_j)}\Big]$$

$$- i\sum_{j=-2}^{\infty} \frac{e^{i\alpha_j|x-x_*|}}{\cosh\,(\alpha_j H)K_2'(\alpha_j)}, \tag{2.15}$$

$$\eta(x) = -i\sum_{k=0}^{\infty} \frac{e^{-i\gamma_k x}\,K_+(\gamma_k)}{K_1'(\gamma_k)}\Big[a + b\gamma_k - \sum_{m=-2}^{\infty} \frac{e^{i\alpha_m x_*}(\beta\alpha_m^4 - d)}{\cosh\,(\alpha_m H)K_2'(\alpha_m)K_+(\alpha_m)(\alpha_m - \gamma_k)}\Big].$$

For $x_* < 0$, the analogous formulas have the form

$$w(x) = -i\sum_{j=-2}^{\infty} \frac{e^{i\alpha_j x}\,\alpha_j \tanh\,(\alpha_j H)}{K_+(\alpha_j)K_2'(\alpha_j)}\Big[a - b\alpha_j + \sum_{k=0}^{\infty} \frac{e^{-i\gamma_k x_*}(\beta\gamma_k^4 - d)K_+(\gamma_k)}{\cosh\,(\gamma_k H)K_1'(\gamma_k)(\gamma_k - \alpha_j)}\Big],$$

$$\eta(x) = -i\sum_{k=0}^{\infty} \frac{e^{-i\gamma_k x}\,K_+(\gamma_k)}{K_1'(\gamma_k)}\Big[a + b\gamma_k + \sum_{m=0}^{\infty} \frac{e^{-i\gamma_m x_*}(\beta\gamma_m^4 - d)K_+(\gamma_m)}{\cosh\,(\gamma_m H)K_1'(\gamma_m)(\gamma_m + \gamma_k)}\Big] - i\sum_{k=0}^{\infty} \frac{e^{i\gamma_k|x-x_*|}}{\cosh\,(\gamma_k H)K_1'(\gamma_k)}. \tag{2.16}$$

The second sum in expressions (2.15) and (2.16) represents the perturbation from the source, and first sum the perturbation reflected from the edge.

3. Solution Ignoring the Inertial Term. According to the above assumptions, $d \ll 1$. Therefore, in Eq. (1.5), the parameter d can be ignored. For $d = 0$, the solution of the problem can be obtained in explicit form, namely:

$$A_{11} = K_+'(0), \qquad A_{12} = A_{21} = K_+(0), \qquad A_{22} = 0, \qquad C_1 = -(K_+(0)L_-(0))', \qquad C_2 = -K_+(0)L_-(0).$$

Then, we have

$$a = -L_-(0) = \begin{cases} \beta\displaystyle\sum_{j=-2}^{\infty} \frac{e^{i\alpha_j x_*}\,\alpha_j^3}{\cosh\,(\alpha_j H)K_2'(\alpha_j)K_+(\alpha_j)}, & x_* > 0, \\ -\beta\displaystyle\sum_{k=0}^{\infty} \frac{e^{-i\gamma_k x_*}\,K_+(\gamma_k)\gamma_k^3}{\cosh\,(\gamma_k H)K_1'(\gamma_k)}, & x_* < 0, \end{cases}$$

$$b = -L_-'(0) = \begin{cases} \displaystyle\sum_{j=-2}^{\infty} \frac{e^{i\alpha_j x_*}\,\alpha_j^2}{\cosh\,(\alpha_j H)K_2'(\alpha_j)K_+(\alpha_j)}, & x_* > 0, \\ \displaystyle\sum_{k=0}^{\infty} \frac{e^{-i\gamma_k x_*}\,K_+(\gamma_k)\gamma_k^2}{\cosh\,(\gamma_k H)K_1'(\gamma_k)}, & x_* < 0. \end{cases}$$

For $x_* > 0$, we obtain

$$w(x) = i\beta\sum_{j=-2}^{\infty} \frac{e^{i\alpha_j x}\,\alpha_j^3 \tanh\,(\alpha_j H)}{K_+(\alpha_j)K_2'(\alpha_j)} \sum_{m=-2}^{\infty} \frac{e^{i\alpha_m x_*}\,\alpha_m^2}{\cosh\,(\alpha_m H)K_2'(\alpha_m)K_+(\alpha_m)(\alpha_m + \alpha_j)} - i\sum_{j=-2}^{\infty} \frac{e^{i\alpha_j|x-x_*|}}{\cosh\,(\alpha_j H)K_2'(\alpha_j)},$$

$$\eta(x) = i\beta \sum_{k=0}^{\infty} \frac{\mathrm{e}^{-i\gamma_k x}\, \gamma_k^2 K_+(\gamma_k)}{K_1'(\gamma_k)} \sum_{m=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_m x_*}\, \alpha_m^2}{\cosh\,(\alpha_m H) K_2'(\alpha_m) K_+(\alpha_m)(\alpha_m - \gamma_k)}.$$

For $x_* < 0$, these formulas have the form

$$w(x) = -i\beta \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j x}\, \alpha_j^3 \tanh\,(\alpha_j H)}{K_+(\alpha_j) K_2'(\alpha_j)} \sum_{k=0}^{\infty} \frac{\mathrm{e}^{-i\gamma_k x_*}\, \gamma_k^2 K_+(\gamma_k)}{\cosh\,(\gamma_k H) K_1'(\gamma_k)(\gamma_k - \alpha_j)},$$

$$\eta(x) = -i\beta \sum_{k=0}^{\infty} \frac{\mathrm{e}^{-i\gamma_k x}\, \gamma_k^2 K_+(\gamma_k)}{K_1'(\gamma_k)} \sum_{m=0}^{\infty} \frac{\mathrm{e}^{-i\gamma_m x_*}\, K_+(\gamma_m)\gamma_m^2}{\cosh\,(\gamma_m H) K_1'(\gamma_m)(\gamma_m + \gamma_k)} - i \sum_{k=0}^{\infty} \frac{\mathrm{e}^{i\gamma_k |x - x_*|}}{\cosh\,(\gamma_k H) K_1'(\gamma_k)}.$$

Let us now consider the general case of vibrations of the bottom. In this case, multiplying the obtained solution by $u(x_*)$ and integrating over x_*, we obtain all values of interest to us. The solution is written as the sum of two terms that correspond to the bottom segments under the plate and outside the plate $w(x) = w_1(x) + w_2(x)$ and $\eta(x) = \eta_1(x) + \eta_2(x)$, where $w_1(x)$ and $\eta_1(x)$ are the complex vibration amplitudes of the plate and the free surface for the bottom vibrations given by

$$w(x, -H_0, t) = \begin{cases} 0, & x < 0, \\ u(x)\,\mathrm{e}^{-i\omega t}, & x > 0, \end{cases}$$

and $w_2(x)$ and $\eta_2(x)$ are the indicated vibration amplitudes for the bottom vibrations given by

$$w(x, -H_0, t) = \begin{cases} u(x)\,\mathrm{e}^{-i\omega t}, & x < 0, \\ 0, & x > 0; \end{cases}$$

$$w_1(x) = i\beta \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j x}\, \alpha_j^3 \tanh\,(\alpha_j H)}{K_+(\alpha_j) K_2'(\alpha_j)} \sum_{m=-2}^{\infty} \frac{A_m \alpha_m^2}{\cosh\,(\alpha_m H) K_2'(\alpha_m) K_+(\alpha_m)(\alpha_m + \alpha_j)} - i \sum_{j=-2}^{\infty} \frac{\tilde{A}_j(x)}{\cosh\,(\alpha_j H) K_2'(\alpha_j)},$$

$$\eta_1(x) = i\beta \sum_{k=0}^{\infty} \frac{\mathrm{e}^{-i\gamma_k x}\, K_+(\gamma_k)\gamma_k^2}{K_1'(\gamma_k)} \sum_{m=-2}^{\infty} \frac{A_m \alpha_m^2}{\cosh\,(\alpha_m H) K_2'(\alpha_m) K_+(\alpha_m)(\alpha_m - \gamma_k)},$$

$$w_2(x) = -i\beta \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j x}\, \alpha_j^3 \tanh\,(\alpha_j H)}{K_+(\alpha_j) K_2'(\alpha_j)} \sum_{k=0}^{\infty} \frac{B_k \gamma_k^2 K_+(\gamma_k)}{\cosh\,(\gamma_k H) K_1'(\gamma_k)(\gamma_k - \alpha_j)},$$

$$\eta_2(x) = -i\beta \sum_{k=0}^{\infty} \frac{\mathrm{e}^{-i\gamma_k x}\, \gamma_k^2 K_+(\gamma_k)}{K_1'(\gamma_k)} \sum_{m=0}^{\infty} \frac{B_m K_+(\gamma_m)\gamma_m^2}{\cosh\,(\gamma_m H) K_1'(\gamma_m)(\gamma_m + \gamma_k)} - i \sum_{k=0}^{\infty} \frac{\tilde{B}_k(x)}{\cosh\,(\gamma_k H) K_1'(\gamma_k)},$$

where

$$\tilde{A}_j(x) = \int_0^{\infty} \mathrm{e}^{i\alpha_j |x - x_*|}\, u(x_*)\, dx_*, \qquad A_j = \int_0^{\infty} \mathrm{e}^{i\alpha_j x_*}\, u(x_*)\, dx_*,$$

$$\tilde{B}_k(x) = \int_{-\infty}^{0} \mathrm{e}^{-i\gamma_k |x - x_*|}\, u(x_*)\, dx_*, \qquad B_k = \int_{-\infty}^{0} \mathrm{e}^{-i\gamma_k x_*}\, u(x_*) dx_*.$$

4. Numerical Results. Numerical calculations were performed for a semi-infinite ice plate in the ocean for the following parameter values: $E = 6 \cdot 10^9$ N/m^2, $\rho = 1025$ kg/m^3, and $\rho_0 = 922.5$ kg/m^3. The displacements of the bottom were specified as [9]

$$u(x) = \cos^2\,(\pi(x - x_0)/(2s)),$$

where s is the half-width and x_0 is the center of the vibrating segment of the bottom. The plate thickness, fluid depth, and the frequency, center, and half-width of the vibrating segment of the bottom were varied.

Of the greatest interest is the case where the vibrating segment of the bottom is under the plate. Figure 1 shows curves of the outgoing-wave amplitudes in the fluid and in the plate and the plate-vibration amplitudes at the edge and at the point x_0 (epicenter) versus frequency for the following parameter values: $x_0 = 1500$ m, $s = 250$ m, $h = 5$ m, and $H_0 = 200$ m. It is seen from the figure that the maximum vibration amplitudes are observed for

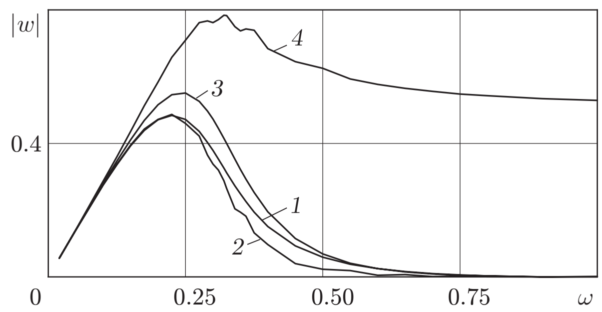

Fig. 1

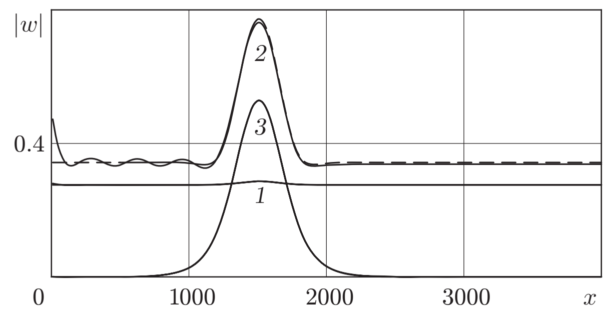

Fig. 2

Fig. 1. Outgoing-wave amplitudes in the fluid (curve 1) and in the plate (curve 2) and the plate-vibration amplitude at the edge (curve 3) and epicenter (curve 4) versus frequency.

Fig. 2. Plate-vibration amplitudes for various frequencies: curves 1–3 correspond to values of 0.1, 0.3, and 1 $\sec^{-1}$, respectively.

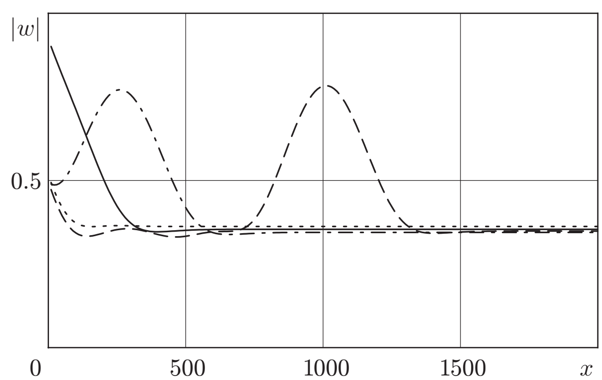

Fig. 3

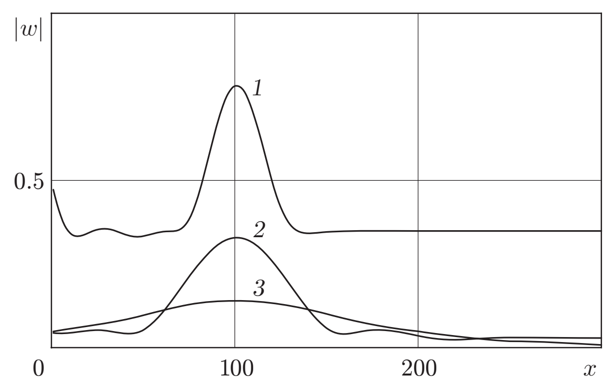

Fig. 4

Fig. 3. Effect of the position of the vibrating bottom segment on the vibration amplitudes: the solid curve refers to $x_0 = 0$, the dashed curve to $x_0 = 1000$ m, the dot-and-dashed curve to $x_0 = 250$ m, and the dotted curve to $x_0 = -1000$ m.

Fig. 4. Effect of fluid depth on the plate-vibration amplitude: curves 1–3 refer to depths of 200, 500 and 1000 m, respectively.

$\omega = 0.25$–$0.3 \sec^{-1}$. As the frequency increases, all amplitudes decrease but the amplitude at the epicenter remains rather large.

Figure 2 shows curves of the plate-vibration amplitudes for various frequencies and the same values of the remaining parameters (the dashed curve corresponds to an infinite plate). For low frequencies, the dimensionless parameter β is small, the plate behaves as the free surface of the fluid, and the plate-vibration amplitudes are almost constant. As the frequency increases, the vibration amplitude becomes much larger at the center than at the remaining points. For high frequencies, only the neighborhood of the vibrating bottom segment vibrates and the vibration amplitudes of the remaining part of the plate are small. The edge effect is insignificant and is manifested only near the edge of the plate.

Figure 3 gives curves of the plate-vibration amplitudes for $\omega = 0.3 \sec^{-1}$, $s = 250$ m, $h = 5$ m, $H_0 = 200$ m, and various positions of the vibrating segment of the bottom. As is evident from the figure, the maximum vibration amplitudes are reached above the center of the vibrating bottom segment when it is located under the plate edge.

As the depth increases, the plate-vibration amplitudes decrease, as is evident from Fig. 4, which gives curves of the plate-vibration amplitudes for $\omega = 0.3 \sec^{-1}$, $x_0 = 1000$ m, $s = 250$ m, $h = 5$ m, and various depths. As the depth increases, the curves become flatter.

Figure 5 gives curves of the vibration amplitudes of the fluid and the plate versus plate thickness for $\omega = 0.3 \sec^{-1}$, $x_0 = 750$ m, $s = 250$ m, $h = 5$ m, and $H_0 = 100$ m. When the plate thickness is small, the

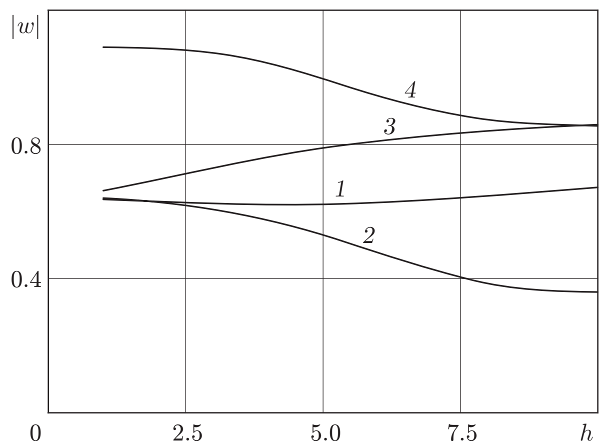

Fig. 5

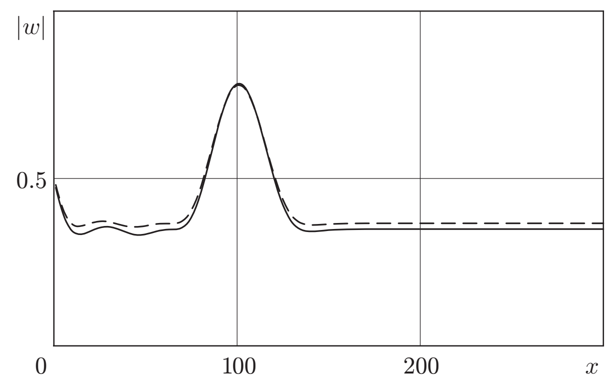

Fig. 6

Fig. 5. Effect of plate thickness on the vibration amplitudes of the fluid and the plate: curves 1 and 2 refer to the outgoing-wave amplitudes in the fluid and plate, respectively, curve 3 to the amplitude of plate deflection at the edge, and curve 4 to the amplitude of plate deflection at the center x_0.

Fig. 6. Comparison of the solution taking into account the plate inertia (solid curve) and the solution ignoring it (dashed curve).

amplitude at the point x_0 is much larger than that at the edge and the amplitudes of the outgoing waves in the fluid and plate are equal. As the thickness increases, the amplitude at the epicenter becomes comparable to the amplitude at the edge and the amplitude of the outgoing wave in the fluid becomes much larger than that in the plate.

Figure 6 shows a comparison of the solution taking into account the plate inertia (solid curve) and the solution ignoring it (dashed curve) for $\omega = 0.3$ sec^{-1}, $x_0 = 1000$ m, $s = 250$ m, $h = 5$ m, and $H_0 = 200$ m. In the figure, the curves are closely spaced. The reason for this is that the inertia of the plate is smaller than that of the fluid. The results obtained show that the explicit solution ignoring the inertia of the plate can be successfully used to estimate the plate deflection.

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